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is doubtful if the very large discordances between the observed and tabular times of the numerous occultations of stars observed between 1680 and 1730 could be similarly eliminated. But there is no such means of removing the discordances which the change would introduce between the observed and tabular places of the Sun during the period 1750–1880, and these discordances are far greater than can be ascribed to the outstanding errors of observation.

Natal Observatory:
1906 *April* 3.

On the Ratios of the triangles in the Determination of the Elliptic Orbit from three Observations. By Shin Hirayama.

1. In this paper I determine the ratios of the triangles which play an important part in the problem of determining the orbit from three observations. Gibbs has determined these ratios by expanding the heliocentric rectangular coordinates of the planet in powers of time, while I expand them in a series of sines and cosines of the mean anomaly and in powers of eccentricity, as is generally done in the elliptic theory.

2. For the three observations of the planet at the times t_1, t_2, and t_3 we have:

$$t_3-t_2=\tau_1. \qquad t_3-t_1=\tau_2. \qquad t_2-t_1=\tau_3.$$

x_1, y_1, z_1	three rectangular heliocentric coordinates of planet.
x_2, y_2, z_2	
x_3, y_3, z_3	
r_1, r_2, r_3	radii vectores of planet.
n	mean daily motion expressed in seconds of arc.
a	semi-major axis of planet.

3. Let

$$x=a_0+a_1\sin nt+b_1\cos nt+a_2\sin 2nt+b_2\cos 2nt \quad . \quad . \quad (1)$$

where a_0, a_1, b_1, a_2, b_2, are five constants depending on the elements of the orbit. Then the accelerations along the x axis corresponding to the three positions will be represented by the following expressions respectively:

$$\text{For } t=-\tau_3;\quad \frac{d^2x}{dt^2}=-\frac{x_1}{r_1^3}=-n^2a_1\sin n\tau_3+n^2b_1\cos n\tau_3-4n^2a_2\sin 2n\tau_3+4n^2b_2\cos 2n$$

$$\text{For } t=0\quad;\quad \frac{d^2x}{dt^2}=-\frac{x_2}{r_2^3}=\qquad n^2b_1\qquad\qquad +4n^2b_2$$

$$\text{For } t=+\tau_1;\quad \frac{d^2x}{dt^2}=-\frac{x_3}{r_3^3}=+n^2a_1\sin n\tau_1+n^2b_1\cos n\tau_1+4n^2a_2\sin 2n\tau_1+4n^2b_2\cos 2n$$

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Hence we have

$$
\begin{aligned}
x_1 &= a_0 - a_1 \sin n\tau_3 + b_1 \cos n\tau_3 - a_2 \sin 2n\tau_3 + b_2 \cos 2n\tau_3 \\
x_2 &= a_0 \qquad\qquad\qquad + b_1 \qquad\qquad\qquad\qquad + b_2 \\
x_3 &= a_0 + a_1 \sin n\tau_1 + b_1 \cos n\tau_1 + a_2 \sin 2n\tau_1 + b_2 \cos 2n\tau_1 \\
\frac{x_1}{n^2 r_1{}^3} &= \quad - a_1 \sin n\tau_3 + b_1 \cos n\tau_3 - 4a_2 \sin 2n\tau_3 + 4b_2 \cos 2n\tau_3 \\
\frac{x_2}{n^2 r_2{}^3} &= \qquad\qquad\qquad + b_1 \qquad\qquad\qquad\qquad + 4b_2 \\
\frac{x_3}{n^2 r_3{}^3} &= \quad + a_1 \sin n\tau_1 + b_1 \cos n\tau_1 + 4a_2 \sin 2n\tau_1 + 4b_2 \cos 2n\tau_1
\end{aligned}
$$

From these six equations the five constants a_0, a_1, b_1, a_2, b_2 may be eliminated, leaving an equation of the form

$$
\left.
\begin{aligned}
&\left[4(S_2 \sin n\tau_1 - S_1 \sin 2n\tau_1) + \frac{4S_1 \sin 2n\tau_1 - S_2 \sin n\tau_1}{n^2 r_1{}^3}\right] x_1 \\
&\quad + \left[4(S_2 \sin n\tau_2 - S_1 \sin 2n\tau_2) + \frac{4S_1 \sin 2n\tau_2 - S_2 \sin n\tau_2}{n^2 r_2{}^3}\right] x_2 \\
&\qquad + \left[4(S_3 \sin n\tau_3 - S_1 \sin 2n\tau_3) + \frac{4S_1 \sin 2n\tau_3 - S_2 \sin n\tau_3{}^3}{n^2 r_3{}^3}\right] x_3 = 0
\end{aligned}
\right\} \quad \ldots (2)
$$

where

$$
\begin{aligned}
S_1 &= \sin n\tau_1 + \sin n\tau_2 + \sin n\tau_3 = -4 \sin\frac{n\tau_1}{2} \sin\frac{n\tau_2}{2} \sin\frac{n\tau_3}{2} \\
S_2 &= \sin 2n\tau_1 + \sin 2n\tau_2 + \sin 2n\tau_3 = -4 \sin n\tau_1 \sin n\tau_2 \sin n\tau_3
\end{aligned}
$$

If we put

$$
\left.
\begin{aligned}
S &= \cos\frac{n\tau_1}{2} \cos\frac{n\tau_2}{2} \cos\frac{n\tau_3}{2} \\
P_1 &= 4S - \cos n\tau_1 = 1 + \cos n\tau_2 + \cos n\tau_3 \\
P_2 &= 4S - \cos n\tau_2 = 1 + \cos n\tau_3 + \cos n\tau_1 \\
P_3 &= 4S - \cos n\tau_3 = 1 + \cos n\tau_1 + \cos n\tau_2 \\
Q_1 &= \cos n\tau_1 - S = 3S - P_1 \\
Q_2 &= -\cos n\tau_2 + S = -3S + P_2 \\
Q_3 &= \cos n\tau_3 - S = 3S - P_3
\end{aligned}
\right\} \quad \ldots \quad \ldots (3)
$$

then we easily derive from (2) the following equation:

$$
\begin{aligned}
&\sin n\tau_1 \left(P_1 + \frac{Q_1}{n^2 r_1{}^3}\right) x_1 - \sin n\tau_2 \left(P_2 - \frac{Q_2}{n^2 r_2{}^3}\right) x_2 \\
&\qquad\qquad + \sin n\tau_3 \left(P_3 + \frac{Q_3}{n^2 r_3{}^3}\right) x_3 = 0 \qquad \ldots (4)
\end{aligned}
$$

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Whence we see that the three triangles determined each by a pair of the three radii vectores, and usually denoted by $[r_2 r_3]$, $[r_1 r_3]$, $[r_1 r_2]$, are respectively proportional to

$$\left(P_1+\frac{Q_1}{n^2 r_1^3}\right)\sin n\tau_1,\quad \left(P_2-\frac{Q_2}{n^2 r_2^3}\right)\sin n\tau_2,\quad \left(P_3+\frac{Q_3}{n^2 r_3^3}\right)\sin n\tau_3 \qquad (5)$$

4. Expand S, P_1, Q_1, &c. in powers of τ, and neglect the higher powers of τ. Substituting all these values in (4), and after dividing by common factors, we get the equation

$$\tau_1\left(1+\frac{\tau_2\tau_3-\tau_1^2}{12 r_1^3}\right)x_1-\tau_2\left(1-\frac{\tau_1\tau_3+\tau_2^2}{12 r_2^3}\right)x_2+\tau_3\left(1+\frac{\tau_1\tau_2-\tau_3^2}{12 r_3^3}\right)x_3=0$$

which is the equation first obtained by Gibbs.

5. In the above explained method for the determination of the ratios of triangles we assume a suitable value of n, and compute these ratios by (3) and (5). We find thus a very close approximation to their correct values if the eccentricity of the ellipse be small. The starting equation (1) shows that there we have already neglected certain terms of the squares of eccentricity.

6. To test the accuracy of the preceding formulæ let us take the *Ceres* example treated by Gauss. The ratios of triangles are computed for four different values of n; while the rigorous values of $[r_2 r_3]$, $[r_1 r_3]$, and $[r_1 r_2]$ are computed by the expressions $r_2 r_3 \sin(v_3-v_2)$, $r_1 r_3 \sin(v_3-v_1)$, and $r_1 r_2 \sin(v_2-v_1)$ respectively. The results are shown in the following table, the corresponding values from Oppolzer and Gibbs being added for comparison.

		$\log\frac{[r_2 r_3]}{[r_1 r_3]}$.	Error.	$\log\frac{[r_1 r_2]}{[r_1 r_3]}$.	Error.
Rigorous Formula		9·751242	0	9·776875	0
Oppolzer's „		9·742730	+8512	9·768466	+8409
Gibbs' „		9·751316	− 74	9·776953	− 78
Hirayama's Formula n =	1000″	9·751122	+ 120	9·777070	− 95
	769·6845	9·751280	+ 38	9·776859	+ 16
	750	9·751287	− 45	9·776853	+ 22
	500	9·751323	− 81	9·776864	+ 9

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On the Orbit and Relative Masses of β 733 (85 *Pegasi*).

By W. Bowyer and H. H. Furner.

β 733.

	h m s	
R.A.	23 56 57	1900 Mags. 6 : 10
N.P.D.	63° 27′	

The discussion of the meridian observations of 85 *Pegasi* for a determination of the relative masses, incidentally showed that the period of this binary did not exceed twenty-five years.

The orbit has been computed at various times, the results so far as the period is concerned being—

	Years.
1888 Schaeberle...	22·3
1892 Glasenapp	17·5
1895 See...	24·0
1899 Burnham	25·7

This failure to secure harmony has led to a discussion of the micrometer measures, which are best satisfied by a period of twenty-six years—*i.e.* a little larger than that deduced by Professor Burnham.

The meridian observations extend over a period of seventy-five years, or three revolutions, and the period given can be assumed to be more accurate than that at present derived from the micrometer measures. One explanation of the discrepancy may be found in the difficulty of measuring so close and dissimilar a pair; but there is a tendency to periods of alternate positive and negative residuals when comparing observed and computed places which would point to some real disturbing cause. This would show to a greater extent in the micrometer measures than in the meridian observations.

In Fig. 1 all available measures have been taken in groups, plotted down, and the apparent ellipse drawn. From this the elements of the true ellipse have been found to be:—

$e = \cdot 46$ $\quad\quad$ $a = 0''\cdot 82$

$☊ = 115^{\circ}\ 38'$ $\quad\quad$ $T = 1883\cdot 5$

$\gamma = 53\ \ 5$ $\quad\quad$ $P = 26\cdot 3$ years

$\lambda = 266\ \ 7$

From the apparent ellipse the relative movement of the two components is found to be 1″·35 in right ascension and 1″·05 in